Statistics

Lecture 14

Feb 19-8:47 AM

It has been reported that less than 30% of all LA residents are a supporter of the olympic being held in LA in 2028.

$P < .3$ claim
$\uparrow$
H_1

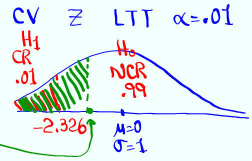
I took a survey of 285 LA residents and 27% of them had same views.

$n = 285 \quad \hat{p} = .27$
$x = n\hat{p}$
$= 285(.27) = 76.95$
$\boxed{x = 77}$

Test the claim at $\alpha = .01$.

$H_0: P \geq .3$
$H_1: p < .3$ claim, LTT

CV $\quad Z \quad$ LTT $\quad \alpha = .01$



$Z = \text{invNorm}(.01, 0, 1)$

$\boxed{\begin{array}{l} \text{CTS} \quad Z = -1.099 \\ \text{P-Value} \quad P = .136 \checkmark \end{array}}$

1 – PropZTest
$P_0 : .3 \qquad H_0$
$x = 77$
$n = 285$
Prop $< P_0 \qquad H_1$
Calculate

CTS is in NCR
P-Value $> \alpha$ $\Rightarrow$ H_0 Valid
H_1 invalid

Invalid claim
Reject the claim

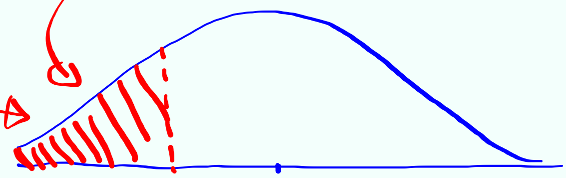
After these findings
$P(H_0 \text{ Valid}) = 1 - \text{P-Value}$
$= 1 - .136$
$= \boxed{.864}$

$P(H_1 \text{ Valid}) = \text{P-Value}$
$= .136$

May 30-8:06 AM

Given CTS $Z = -1.099$, LTT

find p-Value.



-1.099 $\mu = 0$

Area $\sigma = 1$

$= \text{normalcdf}(-E99, -1.099, 0, 1)$

$= \boxed{.136}$

May 30-8:22 AM

City of LA claims the mean rent for all houses in LA with 3BR & 2Bath is \$3800/mo.

$\mu = 3800$ claim

H_0

LA Times Conducted a Survey of 40 Such houses in LA, and mean monthly rent was \$3975. It is Known that standard deviation of monthly rents of all Such houses in LA is \$475.

$n = 40$, $\bar{x} = 3975$, $\sigma = 475$

NO $\alpha \to .05$

Test the claim

σ Known $\to$ Case I

$H_0: \mu = 3800$ claim

$H_1: \mu \neq 3800$ TTT

CV Z TTT $\alpha = .05$

H_1 CR .025 | H_0 NCR .95 | H_1 CR .025

-1.960 $\mu=0$ $\sigma=1$ 1.960

CTS $Z = 2.330$

P-Value $P = .020$ ✓

$Z = \text{invNorm}(.975, 0, 1)$

Z-Test

inpt: Stats

$\mu_0 : 3800$ H_0

$\sigma : 475$

$\bar{x} = 3975$

$n = 40$

$\mu \neq \mu_0$ H_1

CTS is in CR

P-Value $\leq \alpha$ $\Rightarrow$ H_0 invalid

H_1 Valid

Invalid claim

Reject the claim

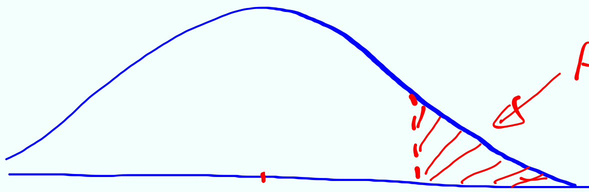
If we choose $\alpha = .01$, then

P-Value $> \alpha \to H_0$ Valid $\to$ Valid claim

$\to$ FTR the claim

May 30-8:25 AM

CTS $Z = 2.330$, TTT find P-Value



Area $\cdot 2$ =

$\boxed{.020}$

$2 \cdot \text{normalcdf}(2.330, E99, 0, 1) =$

May 30-8:42 AM

LAUSD claims the mean of all SAT Scores in the district is above 1200. $\mu > 1200$ (H_1)

I randomly selected 20 SAT exams, the mean was 1300 with standard deviation of 120.

$n=20$, $\bar{x}=1300$, $S=120$

use $\alpha = .02$ to test the claim.

H_0: $\mu \le 1200$

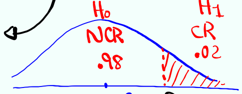
H_1: $\mu > 1200$ claim, RTT

σ unknown → Case II

CV t RTT

$\alpha = .02$

$df = n-1 = 19$



$t = \text{invT}(.98, 19) =$

$\boxed{\begin{array}{l} \text{CTS } t = 3.727 \\ \text{P-Value } p = 7.1 \times 10^{-4} \end{array}}$

T-Test

inpt: $\boxed{\text{Stats}}$

$\mu_0 = 1200$ H_0

$\bar{x} = 1300$

$S = 120$

$n = 20$

$\mu > \mu_0$ H_1

$\boxed{\text{Calculate}}$

CTS is in CR
P-Value $\le \alpha$ ⇒ H_0 invalid, H_1 Valid

Valid claim

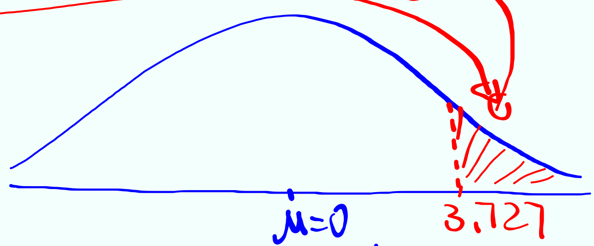
FTR the claim

May 30-8:46 AM

CTS $t = 3.727$ RTT $df = 19$

find P-Value.

Area



$tcdf(3.727, E99, 19) = \boxed{7.1 \times 10^{-4}}$

σ unknown
$df = 19$

May 30-9:00 AM

I randomly selected 12 exams, here are the Scores

84	96	75	100
68	80	70	90
55	88	77	99

1) Find $\bar{x}$ & s. Round to whole #.

$\bar{x} \approx 82$, $s \approx 14$

2) Use $\boxed{\alpha = .1}$ to test the claim that the mean of all exams is 85.

$H_0: \mu = 85$ claim

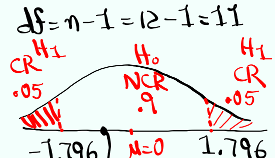
$H_1: \mu \neq 85$ TTT

Since σ is unknown
Case II

CTS $t = -.742$
P-Value $P = .473$

CV t TTT $\alpha = .1$

$df = n-1 = 12-1 = 11$



σ unknown
$df = 11$

T-Test

Stats

inpt:
$\mu_0: 85$
$\bar{x} = 82$
$s = 14$
$n = 12$
$\mu \neq \mu_0$

CTS is in NCR
P-Value > α
H_0 valid
H_1 invalid
H_1

$t = invT(.95, 11)$

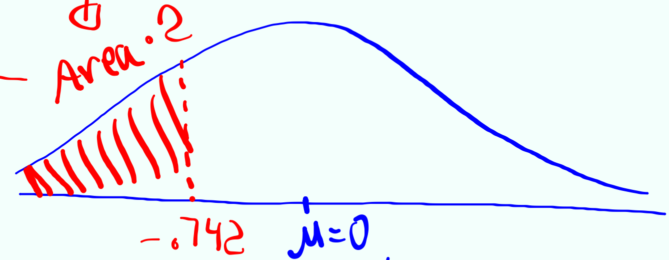
Valid claim

FTR the claim

May 30-9:03 AM

CTS $t = -.742$ TTT $df = 11$

Find p-Value.



σ unknown

$df = 11$

$2 \cdot \text{tcdf}(-E99, -.742, 11)$

$= \boxed{.474}$

SG 25

May 30-9:16 AM

Testing One Population Standard deviation:

$H_0: \sigma = \sigma_0$	$H_0: \sigma \le \sigma_0$	$H_0: \sigma \ge \sigma_0$
$H_1: \sigma \ne \sigma_0$	$H_1: \sigma > \sigma_0$	$H_1: \sigma < \sigma_0$
TTT	RTT	LTT

P-Value Method only

CTS $\chi^2 = \dfrac{(n-1)\cdot S^2}{\sigma^2}$ $df = n-1$

RTT $\chi^2\text{cdf}(CTS, E99, df)$

LTT $\chi^2\text{cdf}(0, CTS, df)$

TTT Find both areas on each side of CTS, Multiply Smaller area by 2.

P-Value $> \alpha \Rightarrow H_0$ Valid, H_1 invalid

P-Value $\le \alpha \Rightarrow H_0$ invalid, H_1 Valid

Final conclusion must be about claim

Reject the claim OR FTR the claim

May 30-9:34 AM

Given: $\boxed{H_0: \sigma \le 10}$, $\boxed{\text{Claim is } H_0}$ $\boxed{\alpha = .02}$

$S = 12$, $n = 8$

Test the claim.

$H_0: \sigma \le 10$ claim

$H_1: \sigma > 10$ RTT

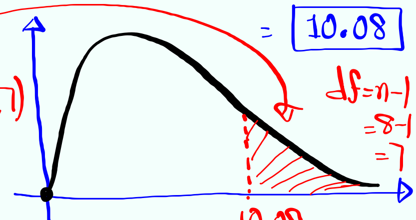
CTS

$$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(8-1)\cdot 12^2}{10^2} = \boxed{10.08}$$

$df = n-1 = 8-1 = 7$

P-Value $= \chi^2 cdf(10.08, E99, 7)$

$= \boxed{.184}$



P-Value $>$ α

.184 .02

H_0 Valid $\rightarrow$ Valid claim

H_1 invalid

FTR the claim

May 30-9:41 AM

AAA claims that standard deviation of Speed of all Cars on the FWY 10 is at least 8 mph.

$\sigma \ge 8$ claim
$\uparrow H_0$

I took 10 Cars, standard deviation of their Speeds on FWY 10 was 5 mph.

$n = 10$, $S = 5$

No $\alpha \rightarrow .05$

Test the claim.

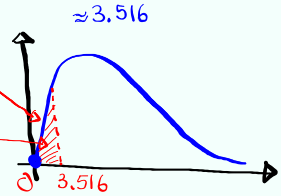
$H_0: \sigma \ge 8$ claim

$H_1: \sigma < 8$ LTT

CTS

$$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(10-1)\cdot 5^2}{8^2}$$

≈ 3.516



Area = P-Value

$\chi^2 cdf(0, 3.516, 9) = \boxed{.060}$

P-Value $>$ α

.060 .05

H_0 valid H_1 invalid

Valid claim

FTR the claim

If we choose α to be .07, .08, .09, .1, - - -,

P-Value $\le \alpha$ H_0 invalid

Invalid claim $\rightarrow$ Reject

May 30-9:50 AM

Math department claims that standard deviation of all math exams is 12.

$\sigma = 12$ ← H_0

I took 10 exams, standard deviation of their scores was 15.

$n = 10$, $S = 15$

$df = n - 1 = 9$

Use $\boxed{\alpha = .1}$ to test the claim.

$H_0: \sigma = 12$ claim

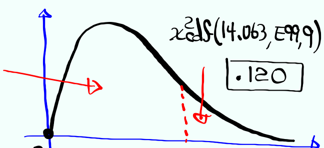
$H_1: \sigma \neq 12$ TTT

CTS

$$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(10-1)\cdot 15^2}{12^2} \approx \boxed{14.063}$$

$\chi^2cdf(0, 14.063, 9) = \boxed{.880}$

$\chi^2cdf(14.063, E99, 9)$ $\boxed{.120}$



0

14.063

P-Value $= 2 \cdot$ Smaller area $= 2(.120) = \boxed{.240}$

P-Value $>$ α

.240 .1

H_0 Valid → Valid claim

H_1 invalid

FTR the claim

May 30-10:01 AM

I randomly selected 10 students, here are their ages:

$n = 10$

$df = 9$

1) find $\bar{x}$ & S.

Round to whole #

25	28	18
30	19	25
36	40	50
	38	

$\bar{x} \approx 31$

$S \approx 10$

2) Test the claim that standard deviation of ages of all students is not 8 yrs.

NO α → .05

$\sigma \neq 8$ claim

↑ H_1

$H_0: \sigma = 8$

$H_1: \sigma \neq 8$ claim, TTT

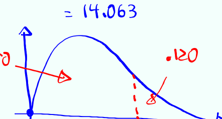
CTS

$$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(10-1)\cdot 10^2}{8^2} = 14.063$$

See last example

.880

.120



14.063

P-Value .240

P-Value $> \alpha$

H_0 Valid

H_1 invalid → Invalid claim → **Reject the claim**

Optional SG 3

May 30-10:12 AM

Comparing two Population Standard deviations:

$H_0: \sigma_1 = \sigma_2$	$H_0: \sigma_1 \le \sigma_2$	$H_0: \sigma_1 \ge \sigma_2$
$H_1: \sigma_1 \ne \sigma_2$	$H_1: \sigma_1 > \sigma_2$	$H_1: \sigma_1 < \sigma_2$
TTT	RTT	LTT

Group 1	Group 2
n_1	n_2
S_1	S_2

$S_1 > S_2$

CTS $F = \dfrac{S_1^2}{S_2^2}$

$Ndf = n_1 - 1$, $Ddf = n_2 - 1$

RTT
P-Value = fcdf(CTS, E99, Ndf, Ddf)

LTT
P-Value = fcdf(0, CTS, Ndf, Ddf)

TTT
Find area on both sides of CTS, multiply smaller area by 2.

Using TI Command
2-SampFTest
for
CTS F & P-Value p

Proceed with P-Value Method.

May 30-10:37 AM

Consider the chart below

Group 1	Group 2
$n_1 = 8$	$n_1 = 10$
$S_1 = 10$	$S_2 = 5$

1) Verify $S_1 > S_2$ ✓

2) CTS $F = \dfrac{S_1^2}{S_2^2} = \dfrac{10^2}{5^2} = 4$ ✓

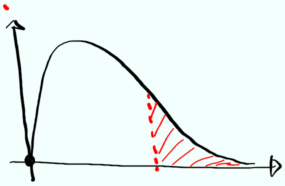
$Ndf = n_1 - 1 = 7$

$Ddf = n_2 - 1 = 9$

3) Test the claim $\sigma_1 > \sigma_2$.

$H_0: \sigma_1 \le \sigma_2$

$H_1: \sigma_1 > \sigma_2$ claim RTT



P-Value ≤ α
.029 .05

H_0 invalid, H_1 Valid
Valid claim
FTR

P-Value = fcdf(4, E99, 7, 9)
$= \boxed{.029}$

$\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$ $\boxed{\text{2-SampFTest}}$

CTS F = 4
P-Value p = .029

May 30-10:46 AM

Consider the chart below

Group 1	Group 2
$n_1 = 10$	$n_2 = 10$
$S_1 = 6$	$S_2 = 5$

1) Is $S_1 > S_2$ ✓

2) CTS $F = \frac{S_1^2}{S_2^2} = \frac{6^2}{5^2} = 1.44$

$Ndf = n_1 - 1 = 9$

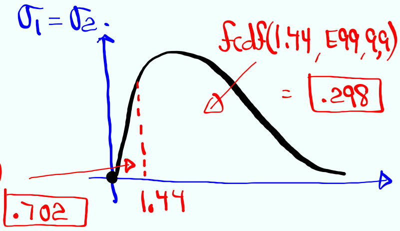
$Ddf = n_2 - 1 = 9$

3) Test the claim $\sigma_1 = \sigma_2$.

$H_0: \sigma_1 = \sigma_2$ claim

$H_1: \sigma_1 \neq \sigma_2$ TTT

Fcdf(0, 1.44, 9, 9) $= \boxed{.702}$



Fcdf(1.44, E99, 9, 9) $= \boxed{.298}$

P-Value $>$ α

.596 .05

H_0 Valid

H_1 invalid

Valid claim

FTR

P-Value = 2 · Smaller area

$= 2 \cdot .298 = \boxed{.596}$

use 2-SampFTest

CTS F = 1.44

P-Value p = .596

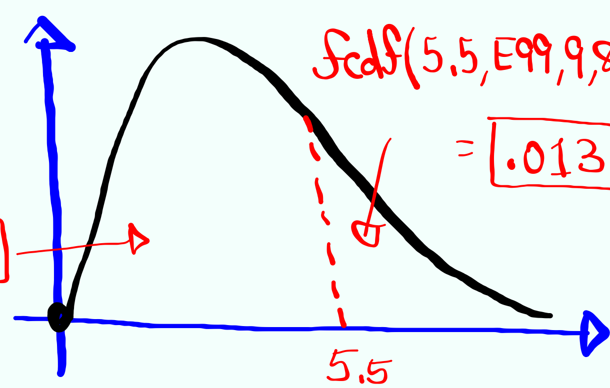
May 30-10:56 AM

Given CTS F = 5.5, Ndf = 9, Ddf = 8

TTT

Find P-Value

Fcdf(0, 5.5, 9, 8) $= \boxed{.987}$



Fcdf(5.5, E99, 9, 8) $= \boxed{.013}$

P-Value = 2 · Smaller Area = 2(.013)

$= \boxed{.026}$

You can Start SG 29

May 30-11:07 AM